Classes of prime numbers and their differences

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Abstract

In this paper, the author discusses primes' and their differences' classes on the base of 6. These classes are of importance in distribution values of differences of primes, as the first ones, gaps, as the second ones, variations.

The main reason of six period oscillations in distributions both gaps and variations and the peculiarity of each of them is the distribution of primes themselves in groups of the same class. Besides, in such an oscillation in exponent distribution of variations values, the numbers determining the class of each a variation on the same base play an essential role.

An analytical research of a problem of independent appearing of a next prime of the same class as of a current one or of the opposite class proves it possible in the infinite general set of prime numbers only.

1 Introduction

The assumption that consecutive primes can be of any classes, $P = 6n + 1$ and $P = 6n - 1$, independently only because they are equally likely creates the probability of being both of consecutive primes of the same class(having the gap between them multiple of 6) equal 1/2. But, the statistical analysis in the range until 1,000,000 give this probability equal $\sim 40\%$. The problem of probability of being consecutive primes of the same class equal 1/2 must be researched analytically.

It is of very important role that all primes are distributed in groups of the same class beginning from the size equal 1 (singles) to maximal one in a range of primes. This size increases when increases a range, and a number of primes in a groups of bigger size is less than a number of them in a groups of smaller size. In the range until 100,000,000, the maximal size of such groups is equal 16. Besides, the number of single primes of a class is approximately equal to the number of primes in pairs of the same class.

Groups of the same class create inside them differences of consecutive primes multiple of 6. The first differences, gaps, G^0 exist inside groups of the size more than 1 (non-singles). The second differences, variations, V^0 do inside groups of a size more than 2.

2 Classes of primes and their differences on the base of 6

2.1 The Key of Primes Structure

The number 6 is the key of primes structure because all primes except 2 and 3, that must be considered special, basic, ones, are equal $P = 6m \pm 1$ ($m = 1, 2, 3, \cdots$) ($P = 6n + 1$ or $6n + 5$ [1]).

Primes, gaps, and variations constitute classes on the base of 6. Primes, except 2 and 3, constitute 2 classes: $P^+ = 6m + 1$ and $P^- = 6m - 1$. Gaps and variations constitute 3 classes each: gaps - $G^0 = 6m = 2(3m + 0)$, $G^+ = 2[3(m - 1) + 1]$, and $G^- = 2(3m - 1)$, the same for postgaps; variations - $V^0 = 6a = 2(3a + 0)$, $V^+ = 2(3a + 1)$, $V^- = 2(3a - 1)(a = 0, \pm 1, \pm 2, \pm 3, \ldots)$.

Thus, $\{P\} = \{P^2\} = \{P^{\sim 0}\} = \{P^+, P^-\}$; $\{G\} = \{G^3\} = \{G^0, G^+, G^-\}$; $\{V\} = \{V^3\} = \{V^0, V^+, V^-\}$. Classes are determined by $\{K^k\} = \{0, \{\sim 0\}\} = \{0, +1, -1\}$: $P^k = 6m + K^k$, $G^k = 2(3m + K^k)$, $V^k = 2(3a + K^k)$.

2.2 Distribution of gap and variation values

They can see an essential role of the membership of a class in distribution of gaps and variations values especially. For the distribution of gaps values, it is only that gaps that are multiple of 6 (G^0) are more common than other ones (see Fig. 1).

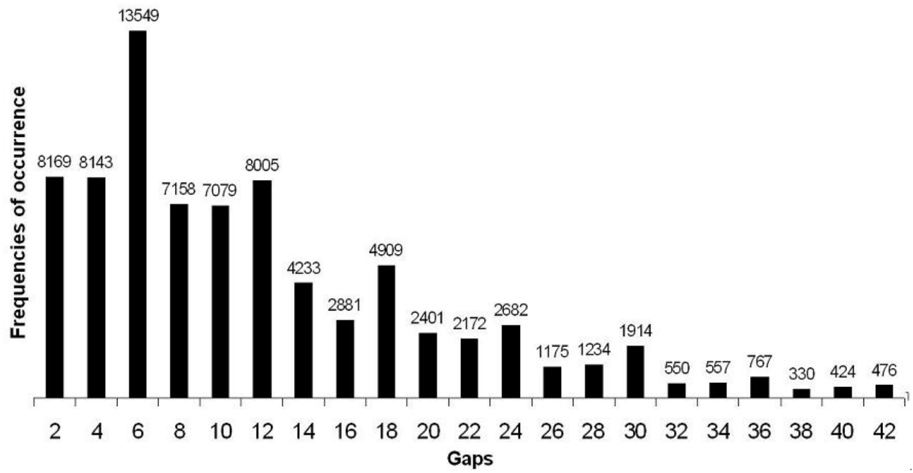


Figure 1: Distribution of gap in the range until 1,000,000.

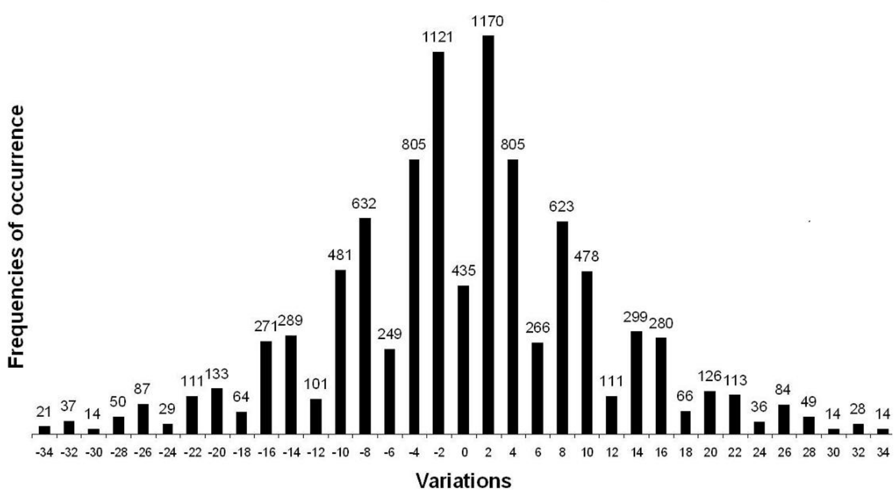


Figure 2: Distribution of variations in the range until 100,000.

But the rule of the distribution of variation values is much stricter than one of gaps values, (see Fig. 2) Variations $|V|^{\sim 0}$ constitute major series, and ones V^0, multiple of 6, do minor series.

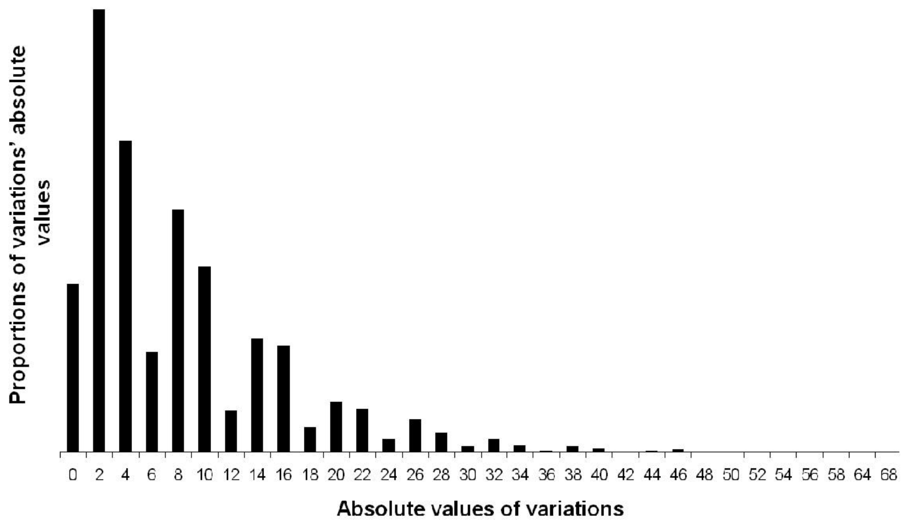


Figure 3: Distribution of proportions of variations absolute values.

For distribution of proportions of variations' absolute values (ratios of frequencies of occurrence to the number of primes in a range) (see Fig. 3) there are such the regressions found with MS Excel (frequencies of occurrence of V=0 are doubled):

Range of primes until	100,000,000	200,000,000
$\|V\|^{\sim 0}$ constitute series	$0.196e^{-0.071\|V\|}$	$0.172e^{-0.067\|V\|}$
$\|V\|^{0}$ constitute series	$0.079e^{-0.071\|V\|}$	$0.069e^{-0.067\|V\|}$
M/μ	2.474	2.498

Table 1: Regressions of distribution of variations' absolute values.

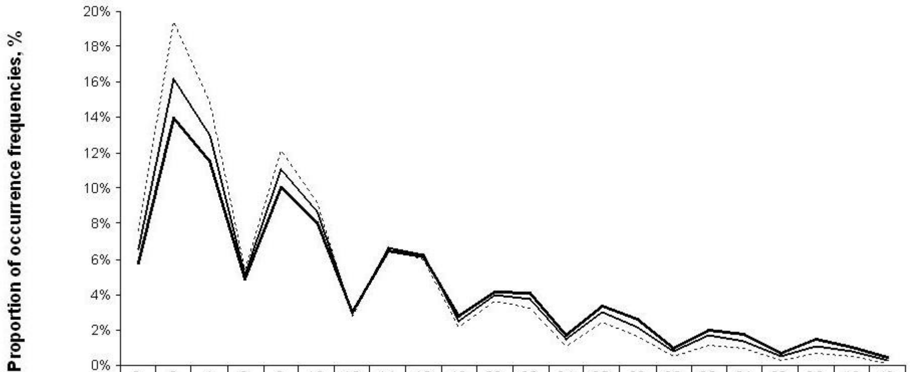
Since $|K| = 1$ for $V^{\sim 0}$ and $|K| = 0$ for V^0 , so

$$Y \approx M(M/\mu)^{|K|-1}e^{-b|V|} \tag{1}$$

In the infinite general set of primes when consecutive primes may be of any class independently (see subsection 3.2)

$$\lim_{P\to\infty} M/\mu = \frac{3}{2}$$

As the range of primes increases, the chart of the distribution of variations' absolute values becomes gentler sloping (Fig. 4)[1].



	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42
------- 10^6	7.63	19.4	14.8	5.37	12.1	9.19	2.73	6.66	5.92	2.17	3.64	3.23	1.10	2.44	1.63	0.48	1.18	0.92	0.29	0.72	0.47	0.11
—— 10^7	6.57	16.1	12.9	5.12	11.0	8.69	2.96	6.63	6.21	2.55	4.03	3.76	1.48	3.02	2.18	0.77	1.68	1.37	0.51	1.12	0.76	0.28
—— 10^8	5.80	13.9	11.5	4.86	10.0	8.09	3.04	6.50	6.18	2.78	4.20	4.09	1.71	3.36	2.58	0.98	2.05	1.75	0.71	1.47	1.04	0.42

Absolute values of variations |V|

Figure 4: Distribution of proportions of variations absolute values.

2.3 Boolean algebra of primes, gaps, and variations classes

The notations of such classes and values determining them:

Kinds of classes

$$3 = \{k\} = \{0, \{\sim 0\}\} = \{0, +, -\} \qquad \{\sim 0\} = \{+, -\}$$

Reverse

$$R(0) = 0, \quad R(+) = -, \quad R(-) = + \qquad R(k) = R, \quad R(R) = k$$

Determining classes values

$$\{P(mod6)\} = \{K^{\sim 0}\} = \{+1, -1\}$$

$$\{V(mod6), G(mod6), g(mod6)\} = \{2K^k\} = 2 * \{K^k\} = 2 * \{0, +1, -1\}$$

$$K^R = -K^k \qquad 2*2K^k \equiv 2K^R \qquad 3*2K^k \equiv 0 \qquad V^R = -V^k \quad (2)$$

Any combinations of sets of primes, gaps (both pregaps and postgaps), and variations classes can be deduced analytically for primes from 7 (being adjacent to basic prime 3, prime 5 creates exceptions from this algebra). Let examine 3 consecutive primes P_{n-1}, P_n, and P_{n+1}, creating a pregap G_n, a postgap g_n, and a variation V_n (see Tab. 2). From this table is obvious:

P_{n-1}	P_n	P_{n+1}	G_n	g_n	V_n
$k \neq 0$ and $R \neq 0$			$P_n - P_{n-1}$	$P_{n+1} - P_n$	$g_n - G_n$
$\in P^k$ K^k	$\in P^k$ K^k	$\in P^k$ K^k	$\in G^0$ $K^k - K^k = 0$	$\in g^0$ $K^k - K^k = 0$	$\in V^0$ $0 - 0 = 0$
$\in P^k$ K^k	$\in P^k$ K^k	$\in P^R$ $-K^k$	$\in G^0$ $K^k - K^k = 0$	$\in g^R$ $-K^k - K^k = -2K^k$	$\in V^R$ $-2K^k - 0 = -2K^k$
$\in P^R$ $-K^k$	$\in P^k$ K^k	$\in P^k$ K^k	$\in G^k$ $K^k + K^k = 2K^k$	$\in g^0$ $K^k - K^k = 0$	$\in V^R$ $0 - 2K^k = -2K^k$
$\in P^R$ $-K^k$	$\in P^k$ K^k	$\in P^R$ $-K^k$	$\in G^k$ $K^k + K^k = 2K^k$	$\in g^R$ $-K^k - K^k = -2K^k$	$\in V^k$ $-2K^k - 2K^k = -2 * 2K^k$

Table 2: Classes of gaps and variations created by combinations of current prime with previous and next ones.

Primes $\rightarrow$ gaps and variations

$$P_n^{k=\sim 0} \rightarrow G_n^{\sim R} \wedge g_n^{\sim k} \wedge V^3 :$$

$$P_n^k \wedge (P_{n-1}^k \wedge P_{n+1}^k) \rightarrow (G_n^0 \wedge g_n^0) \wedge V_n^0 \qquad P_n^k \wedge (P_{n-1}^R \wedge P_{n+1}^R) \rightarrow (G_n^k \wedge g_n^R) \wedge V_n^k$$

$$P_n^k \wedge (P_{n-1}^k \wedge P_{n+1}^R) \rightarrow (G_n^0 \wedge g_n^R) \wedge V_n^R \qquad P_n^k \wedge (P_{n-1}^R \wedge P_{n+1}^k) \rightarrow (G_n^k \wedge g_n^0) \wedge V_n^R$$

Gaps $\rightarrow$ variations and primes

$$G_n^0 \wedge g_n^0 \rightarrow V_n^0 \wedge P_n^{\sim 0}$$

$$G_n^0 \wedge g_n^{k=\sim 0} \rightarrow V_n^k \wedge P_n^R \qquad G_n^{k=\sim 0} \wedge g_n^0 \rightarrow V_n^R \wedge P_n^k$$

$$G_n^{k=\sim 0} \wedge g_n^R \rightarrow V_n^k \wedge P_n^k \qquad G_n^R \wedge g_n^{k=\sim 0} \rightarrow V_n^R \wedge P_n^R$$

Variations $\rightarrow$ gaps and primes

$$V_n^0 \rightarrow (G_n^0 \wedge g_n^0) \wedge P_n^{k=\sim 0} \wedge (P_{n-1}^k \wedge P_{n+1}^k)$$

$$V_n^{k=\sim 0} \rightarrow Gn^3 \wedge g_n^3 \wedge P_n^{k=\sim 0} \wedge (P_{n-1}^{\sim 0} \wedge P_{n+1}^{\sim 0}) =$$

$$= (G_n^0 \wedge g_n^k) \wedge P_n^R \wedge (P_{n-1}^R \wedge P_{n+1}^k) \vee (G_n^R \wedge g_n^0) \wedge P_n^R \wedge (P_{n-1}^k \wedge P_{n+1}^R) \vee (G_n^k \wedge g_n^r) \wedge P_n^k \wedge (P_{n-1}^R \wedge P_{n+1}^R)$$

Pregaps $\rightarrow$ postgaps $\qquad G_n^0 \rightarrow g_n^3 \qquad G_n^{k=\sim 0} \rightarrow g_n^{\sim k}$
The same for postgaps $\rightarrow$ pregaps.

2.4 Oddsymmetriads

$V_c = 0 \in V^0 \to (P_{c-1} \in P^k \wedge P_c \in P^k \wedge P_{c+1} \in P^k) \wedge (G_c \in G^0 \wedge g_c \in g^0)$

So,

$$V_{c-1} \in V^{\sim k} \qquad and \qquad V_{c+1} \in V^{\sim k}$$

$$V_{c+1} = -V_{c-1} \qquad and \qquad V_{c+1} = R(V_{c-1})$$

But, if

$$V_{c-1} \in V^R \to V_{c+1} \in V^k \to V_{c+1} \notin V^{\sim k}$$

So, it is true only if

$$V_{c-1} \in V^0 \qquad and \qquad V_{c+1} \in V^0$$

Next

$$G_{c-1} \in G^0 \qquad and \qquad g_{c-1} \in g^0$$

The same for the rest of members of oddsymmetriad of a bigger size.
Hence, oddsymmetriads exist in groups of primes of the same type.

$$\forall G \in G^0 \qquad and \qquad \forall g \in g^0 \qquad and \qquad \forall V \in V^0$$

3 Problem of appearing of consecutive primes of any class independently

3.1 Theory

Let

$N(P^k)$ - total number of next primes of the same class in the range of primes;

$N(P^R)$ - total number of next primes of an opposite class in the range of primes;

n - the size of a group of the same class;

m - maximal size of a group in the range;

c_n - total number of primes in the groups of the size in the range of primes;

Γ_n - number of groups of the size in the range of primes;

$$\Gamma_n = \frac{c_n}{n} \tag{3}$$

$\pi(x)$ - total number of primes in the range.

$$\pi(x) = \sum_{n=1}^{m} c_n$$

$p(c_n)$ - portion of c_n

$$p(c_n) = \frac{c_n}{\pi(x)} \tag{4}$$

$D^{(a)}$ - a-th difference of consecutive primes

$D^{(1)} \equiv G$ - the first difference, gap, of consecutive primes

$N(G^0)$ - total number of gaps multiple of 6 in the range of primes;

$$N(G^0) = N(P^k) = \sum_{n=2}^{m}(n-1)\Gamma_n = \sum_{n=1}^{m}(n-1)\Gamma_n \tag{5}$$

$D^{(2)} \equiv V$ - the second difference, variation, of consecutive primes (the difference of consecutive gaps)

$N(V^0)$ - total number of variations multiple of 6 in the range of primes;

As all primes in a group except the last one have the next prime of the same class

$$N(V^0) = \sum_{n=3}^{m}(n-2)\Gamma_n = \sum_{n=1}^{m}(n-2)\Gamma_n - \sum_{n=1}^{2}(n-2)\Gamma_n = \sum_{n=1}^{m}(n-2)\Gamma_n + \Gamma_1 \tag{6}$$

$N(D^{(a)0})$ - total number of a-th differences multiple of 6 in the range of primes;

$$N(D^{(a)0}) = \sum_{n=a+1}^{m}(n-a)\Gamma_n = \Gamma_{a+1} + \sum_{n=a+2}^{m}(n-a)\Gamma_n \tag{7}$$

Because only last members of groups have the next prime of an opposite class

$$N(P^R) = \sum_{n=1}^{m}\Gamma_n \tag{8}$$

In the case consecutive primes may be of any class independently

$$N(P^k) = N(P^R) \tag{9}$$

$$\sum_{n=1}^{m}(n-1)\Gamma_n = \sum_{n=1}^{m}\Gamma_n$$

$$\sum_{n=1}^{m}(n-2)\Gamma_n = 0$$

$$\sum_{n=1}^{m}(n-2)\Gamma_n = N(V^0) - \Gamma_1 = 0$$

$$N(V^0) = \Gamma_1 = c_1 \tag{10}$$

Besides,

$$\begin{aligned} N(G^0) &= \sum_{n=2}^{m}(n-1)\Gamma_n = \Gamma_2 + \sum_{n=3}^{m}(n-1)\Gamma_n \\ &= \Gamma_2 + \sum_{n=3}^{m}(n-2)\Gamma_n + \sum_{n=3}^{m}\Gamma_n = \Gamma_2 + N(V^0) + \sum_{n=3}^{m}\Gamma_n \\ &= \Gamma_2 + \Gamma_1 + \sum_{n=3}^{m}\Gamma_n \end{aligned}$$

$$N(G^0) = \sum_{n=1}^{m} \Gamma_n \tag{11}$$

Those are equality of numbers of multiple of 6: gaps to the total number of primes' groups and variations to the total number of single primes. But, the probabilities of being gaps and variations multiple of 6 when consecutive primes can be of any class independently are $\frac{1}{2}$ and $\frac{1}{4}$ accordingly. So,

$$\sum_{n=1}^{m} \Gamma_n = \frac{1}{2}\pi(x) \qquad\qquad \Gamma_1 = c_1 = \frac{1}{4}\pi(x) \tag{12}$$

In general,

$$N(D^{(a)0}) = \sum_{n=a+1}^{m} (n-a)\Gamma_n = \Gamma_{a+1} + \sum_{n=a+2}^{m} (n-a)\Gamma_n = \frac{1}{2^a}\pi(x) \tag{13}$$

Thus,

$$N(D^{(a-1)0}) = 2N(D^{(a)0}) \tag{14}$$

$$\Gamma_a + \sum_{n=a+1}^{m} (n-a+1)\Gamma_n = 2 \sum_{n=a+1}^{m} (n-a)\Gamma_n$$

$$\begin{aligned} \Gamma_a &= \sum_{n=a+1}^{m} (2n-2a-n+a-1)\Gamma_n \\ &= \sum_{n=a+1}^{m} (n-a-1)\Gamma_n = \sum_{n=a+2}^{m} (n-a-1)\Gamma_n = N(D^{(a+1)0}) \end{aligned}$$

$$\Gamma_a = \frac{1}{2^{a+1}}\pi(x)\Gamma_a \tag{15}$$

But, it is possible only for

$$\lim_{m\to\infty} \sum_{n=1}^{m} \Gamma_n = \Gamma_1 + \Gamma_2 + \Gamma_3 + \ldots = \left(\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots\right)\pi(x) = \frac{1}{2}\pi(x) \tag{16}$$

Because $x \nearrow \rightarrow m \nearrow$, that is possible in the infinite set of primes if a group size in this set may be unlimited. Really, as group sizes increase linearly but numbers of groups of a size decrease exponentially, the contribution of groups of sizes $n > 12$ to these numbers is negligible (see Fig. 5).

$$\sum_{n=3}^{\infty} \Gamma_n \approx \sum_{n=3}^{12} \Gamma_n \tag{17}$$

But 12 is not the final maximal size of groups of the same class because even in the range of primes until 1,000,000 the maximal size is equal 16

$$\exists(m = 16 > 12)$$

Really, the portion of next primes of the same class becomes equal almost $\frac{1}{2}$ soon enough - from the range of 12 million (see Fig. 6).

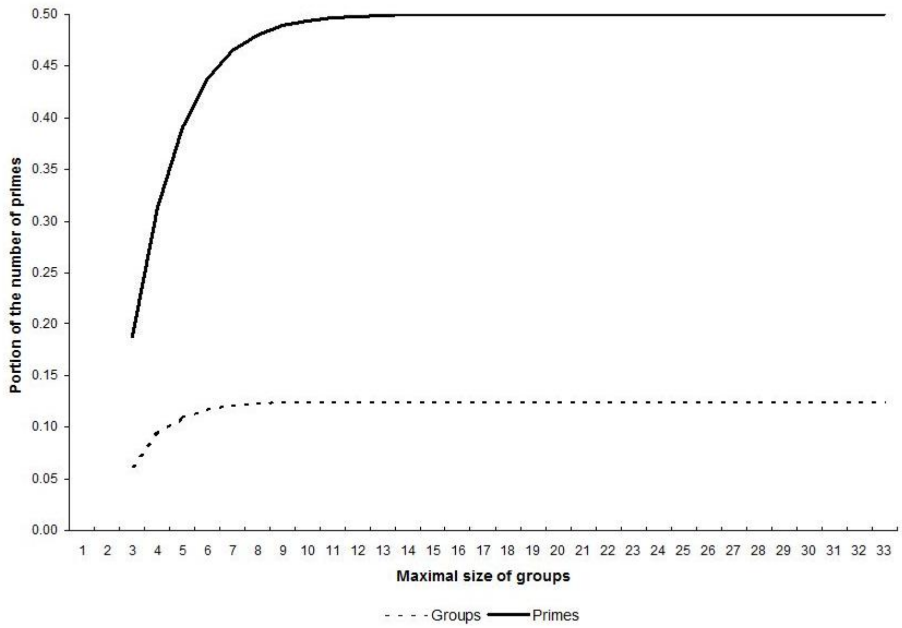


Figure 5: Chart of portions of groups of size ≥ 3 and primes in them

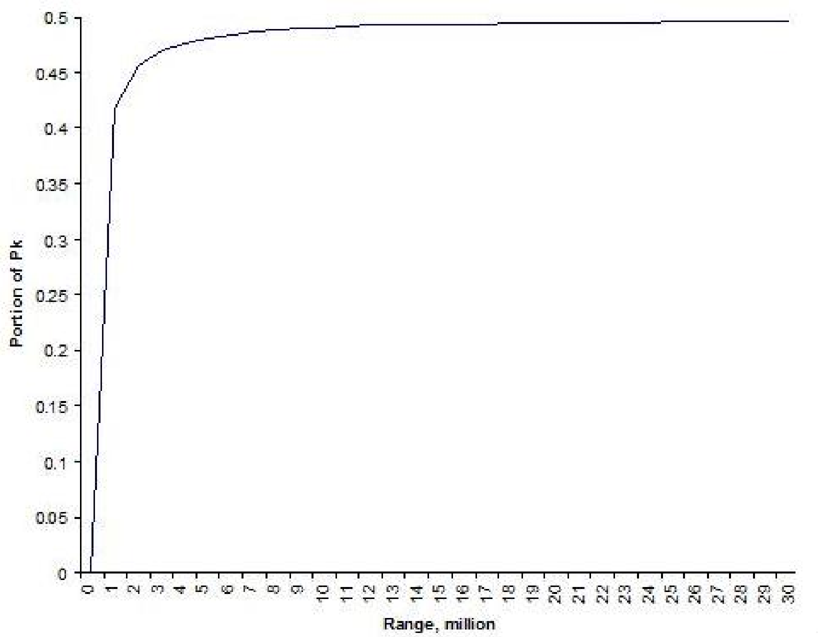


Figure 6: Chart of portions of next primes of the same class

3.2 Extention to the infinite general range of primes

In this range there must be

$$\Gamma_1 = \frac{1}{4}\pi(x),\ \Gamma_2 = \frac{1}{8}\pi(x),\ \sum_{n=3}^{\infty}\Gamma_n = \frac{1}{8}\pi(x) \tag{18}$$

$$c_1 = \frac{1}{4}\pi(x),\ c_2 = \frac{1}{4}\pi(x),\ \sum_{n=3}^{\infty}c_n = \frac{1}{2}\pi(x) \tag{19}$$

This means $\frac{1}{4}$ of P^k have no adjacent P^k.

The number of single P^k having adjacent single P^R is equal

$$2 \times N(P^k \cap P^R \cap P^k \cap P^R) = 2 \times \frac{1}{2^4}\pi(x) = \frac{1}{2^3}\pi(x) = \frac{1}{8}\pi(x) \tag{20}$$

That is $\frac{1}{2}$ of single P^k.

The number of the first and last members of groups of not single P^k is equal

$$2 \times \sum_{n=2}^{\infty}\Gamma_n = 2 \times \frac{1}{4}\pi(x) = \frac{1}{2}\pi(x) \tag{21}$$

The number of P^k inside groups of more than two consecutive P^k is equal

$$\sum_{n=3}^{\infty}c_n - 2 \times \sum_{n=3}^{\infty}\Gamma_n = \frac{1}{2}\pi(x) - 2 \times \frac{1}{8}\pi(x) = \frac{1}{4}\pi(x) \tag{22}$$

Besides, the ratio of coefficients for V^0 and $V^{\sim 0}$ in the formula of distribution of proportions of variations' absolute values for the infinite set of primes is

$$\lim_{x\to\infty} M/\mu = \frac{1 - p(V^0)}{2p(V^0)} = \frac{1 - \frac{1}{4}}{2 \times \frac{1}{4}} = \frac{3}{2} \tag{23}$$

3.3 Statistics in a limited range of primes

In the range of primes until 1,000,000, the portions of primes of the same class on the base of 6 in groups of the sizes are: singles - 32.65%($\sim 1/3$), in pairs - 30.63%($\sim 1/3$), and in all groups of sizes $n \geq 3$ - 36.72%($\sim 1/3$) (see Tab. 3).

$\sim \frac{1}{3}$ primes of a class go single (more than in the infinite general set),

$\sim \frac{1}{3}$ ones go in pairs (more than in the infinite general set),

and $\sim \frac{1}{3}$ ones go in groups of size > 2 (less than in the infinite general set).

Thus,

$$p(c_1) \approx p(c_2) \approx p\left(\sum_{n=3}^{11} c_n\right)$$

$\sim \frac{1}{6}$ of primes go inside groups of more than 2 consecutive P^k (have V^0);

$\sim \frac{1}{2}$ of P^k appear as the first or last members of non single groups,

Size of group	Groups	Primes	Portion, %	$N(V^0)$
Singles	25626	25626	32.65	
Pairs	12023	24046	30.63	
3	4988	14964		
4	1895	7580		
5	754	3770		
6	268	1608		
7	81	567	36.72	12773
8	24	192		
9	9	81		
10	5	50		
11	1	11		

Table 3: Distribution P^k in groups in the range until 1,000,000.

and $\sim \frac{1}{2}$ of them have an adjacent single P^R (found by sorting primes by class [2]).

$$N(V^0) \approx \frac{1}{6}\pi(x) \approx \frac{1}{2}c_1$$

Because only last members of groups have the next prime of an opposite class

$$\begin{aligned} N(P^R) &= \sum_{n=1}^{m}\Gamma_n = \sum_{n=1}^{m}\frac{c_n}{n} = c_1 + \frac{c_2}{2} + \frac{c_3}{3} + \cdots + \frac{c_m}{m} \\ &= \left(\frac{c_1}{2} + \frac{c_2}{2} + \frac{c_3}{2} + \cdots + \frac{c_m}{2}\right) + \left(\frac{1}{2}c_1 - \frac{1}{2\times 3}c_3 - \cdots - \frac{m-2}{2m}c_m\right) \\ &= \frac{1}{2}\pi(x) + \frac{1}{2}\left(\Gamma_1 - \Gamma_3 - \cdots - (m-2)\Gamma_m\right) \end{aligned}$$

$$N(P^R) = \frac{1}{2}\pi(x) - \frac{1}{2}\sum_{n=1}^{m}(n-2)\Gamma_n \tag{24}$$

Correction value to $\frac{1}{2}\pi(x)$ is

$$\frac{1}{2}\left(c_1 - \sum_{n=3}^{m}(n-2)\Gamma_n\right) = \frac{1}{2}\left(c_1 - \sum_{n=3}^{m}\frac{n-2}{n}c_n\right) \tag{25}$$

Besides

$$c_1 - \frac{m-2}{m}\sum_{n=3}^{m}c_n < c_1 - \sum_{n=3}^{m}\frac{n-2}{n}c_n < c_1 - \frac{1}{3}\sum_{n=3}^{m}c_n \tag{26}$$

For this range

$$p(c_1) - \frac{1}{3}\sum_{n=3}^{11}p(c_n) \approx \frac{1}{3} - \frac{1}{3}\times\frac{1}{3} = \frac{2}{9} > 0$$

$$p(c_1) - \frac{11-2}{11}\sum_{n=3}^{11} p(c_n) \approx \frac{1}{3} - \frac{9}{11} \times \frac{1}{3} = \frac{2}{33} > 0$$

Therefore,

$$p(P^R) > \frac{1}{2}$$

It must be

$$\frac{1}{2}\left(1+\frac{2}{33}\right) \approx 0.53 < p(P^R) < \frac{1}{2}\left(1+\frac{2}{9}\right) \approx 0.61$$

. In this range, really, $p(P^R) \approx 0.58$ and $p(P^k) \approx 0.42$.

$$p(P^k) < \frac{1}{2}$$

4 Conclusion

Primes are of 2 nonzero classes on the base of 6. But, their differences as the first ones, gaps, as the second ones, variations, are of the zero class, too,and thus are of 3 classes on this base.

The main reason of six period oscillations in distributions both gaps and variations and the peculiarity of each of them is the distribution of primes themselves in groups of the same class on the base of 6. Besides, in such an oscillation in exponent distribution of variations values, the numbers determining the class of each a variation on the same base play an essential role.

Appearing of consecutive primes of any class independently is possible when $x = \infty$ $(m = \infty)$, i.e. for the infinite general primes' set only. The condition $N(P^k) = N(P^R) = \frac{1}{2}\pi(x)$ approaches to true as $x \to \infty$ but reaches it never. However, it becomes almost true very soon - from the range of numbers of 12 million.

The reason of different condition in a finite set of primes is hidden in real primes' distribution in groups of the same class.

Singles create next primes of an opposite class only (both gaps and variations, not multiple of 6 only). Pairs do equal numbers of next primes of both the classes (equal numbers of gaps multiple of 6 and not; and variations not multiple of 6, only). And groups of $n > 2$ create more next primes of the same class than ones of an opposite class (more gaps , multiple of 6, than not; and variations multiple of 6 for all inner primes of groups).

The predominance of the role of singles over the role of primes in groups of sizes more than 2 is a reason of being $p(P^R) > p(P^k) \neq \frac{1}{2}$ in a finite set of primes.

Notes

[1]Because of that, a suspicion of the possibility of becoming the slope horizontal and even going upward arises. It means to be something similar to the change of "jumping champion", the most common gap, from 6 to 30 near 1.7427×10^{35} and then from 30 to 210 near 10^{425} about what Andrew Odlyzko, Michael Rubinstein, and Marek Wolf have provided a persuasive argument [3].

References

[1] Trost, Ernst. Primzahlen (Elemente der Mathematik Vom Höheren Standpunkt Aus). Verlag Birkhauser, Basel, 1953.

[2] Chassis, Michael. Dynamic Structure of Prime Numbers Distribution. 2004. <http://muchassis.org/primes_structure/index1.htm>

[3] Steward, Ian. "Jumping Champions." Scientific American Dec. 2000: 106-107.